Final Exam
Chapters 1 to 6

Answer the following questions. *You must show your work to receive full credit.* Be sure to make reasonable simplifications. Indicate your final answer with a $\boxed{\text{box}}$.

1. (5 points) Explain in words what the derivative at a point represents and how we approximate it.

2. (5 points) Explain in words what the definite integral represents and how we approximate it.

3. (5 points) Consider the graph of the function f given below. Determine the signs of its first and second derivative.

4. (5 points) Consider the graph of the function g given below. Determine if the following definite integrals are positive, negative or zero.

$(a) \quad \int_{-10}^{4} g(x)dx \qquad\qquad (b) \quad \int_{-6}^{4} g(x)dx$

5. (5 points) If you invest \$100 in a bank account at an annual interest rate of $r\%$, then after 10 years you will have B dollars, where

$$B = 100(1 + \frac{r}{100})^1 0.$$

Find $\frac{dB}{dr}$. In term of money, what does $\frac{dB}{dr}$ represent?

6. (5 points) Interpret the definite integral $\int_{2005}^{2011} f(t)dt$, where $f(t)$ is the rate at which world population is growing in year t, in billions of people per year.

7. (5 points) Consider the graph of the function f below. Represent the indicated area as a definite integral.

8. (5 points) Evaluate $\int \frac{(\ln z)^2}{z} dz$.

9. (5 points) Evaluate $\int_0^1 6te^{-t^2} dt$.

10. (5 points) Find the unique antiderivative, $F(x)$, of the function $f(x) = \frac{2}{x} + x^7$ such that $F(1) = 1$.

11. (5 points) Find the derivative of the function $g(x) = \ln(x^2 + 3x)$. Indicate which rule you are using. (No work, No credit!)

12. (5 points) Let $y = e^{x \ln x}$. Determine if $y' = e^{x \ln x}(1 + \ln x)$. (No work, No credit!)

Pick 4 of the next 6 questions to answer. Put your answers on the blank pages which follow. Clearly indicate which you would like graded.

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13. (10 points)

(a) Find all critical points and inflection points of the function $f(x) = x^4 - 2ax^2 + b$ assuming that a and b are constant.

(b) Find values of the parameters a and b assuming that f has a critical point at the point $(2, 5)$.

(c) If there is a critical point at $(2, 5)$, where are the inflection points of f?

14. (10 points) Find the global maximum and minimum of the function $g(x) = x^3 - 3x^2 - 9x + 15$ on the interval $-5 \leq x \leq 4$.

15. (10 points) An apple tree produces, on average, 400 kg of fruit each season. However, if more than 200 trees are planted per square km, crowding reduces the yield by 1 kg for each tree over 200.

(a) Express the total yield, y, from one square km as a function of the number of trees on it.

(b) How many trees should a farmer plant on each square km to maximize yield?

16. (10 points) An online seller of knitted sweaters finds that it costs \$35 to make her first sweater. Her cost for each additional sweater goes down at a constant rate until it reaches \$25 for her 100^{th} sweater, and after that it starts to rise again. If she can sell each sweater for \$35, what is the quantity sold that maximizes profit? How much profit will she make?

17. (10 points) A forest fire covers 2000 acres at time $t = 0$. The fire is growing at a rate of $8\sqrt{t}$ acres per hour, where t is in hours. How many acres are covered 24 hours later? How many new acres are covered by the fire in the second day?

18. (10 points) A bar of metal is cooling from 1000°C to room temperature, 20°C. The temperature, T, of the bar t minutes after it starts cooling is given, in °C, by

$$T = 20 + 980e^{-0.1t}.$$

(a) Find the temperature of the bar at the end of one hour.

(b) Find the average temperature of the bar over the first hour.